[Time:3:00 Hrs.] **[Marks: 80]**

Please check whether you have got the right question paper.

N.B: 1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Scientific calculator can be used.

Q.1 **a)** If $a_n \neq 0$ for all but finitely many values of n then prove that the radius of convergence R of $\sum_{n=0}^{\infty} a_n z^n$ is related by $\liminf \left|\frac{a_{n+1}}{a_n}\right| \leq \frac{1}{R} \leq \limsup \left|\frac{a_{n+1}}{a_n}\right|$. In particular, if $\lim \left|\frac{a_{n+1}}{a_n}\right|$ exists, then $\frac{1}{R} = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} |a_n|^{1/n}$. **10**

b) Attempt **any Two** of the following: **10**

i) Prove that a branch of logarithms is analytic and find its derivative. **5**

ii) Find the domain of region of convergence of the following power series: $\sum_{n=1}^{\infty} \left(\frac{iz-1}{3+4i}\right)^n$. **5**

iii) Given a series $\sum_{n=1}^{\infty} z^n(1-z)$. Prove that the series converges for $|z| < 1$ and find its sum. **5**

Q.2 **a)** State and prove the Cauchy Integral formula. **10**

b) Attempt **any Two** of the following: **10**

i) Evaluate $\int_x \frac{e^{2z}}{\left(z-\frac{1}{3}\right)^3} dz$ where x is the rectangle with vertices at $\pm i$ and $1 \pm i$. **5**

ii) Evaluate $\int_x \frac{z^2}{z^4-1} dz$ where x is $|z-1| = 1$ **5**

iii) Evaluate $\int_0^{1+i} (x^2 + iy)\, dz$ along the line $y = x$. **5**

Q.3 **a)** State and prove the Schwarz's lemma. **10**

b) Attempt **any Two** of the following: **10**

i) Determine the nature of singularity of $f(z) = \frac{z-2}{z^2} sin\left(\frac{1}{z-1}\right)$. **5**

ii) State and prove the Minimum Modulus Principle. **5**

iii) Find Laurent Series expansions for $f(z) = \frac{2}{(z-1)(z-2)}$ in the region $|z| < 1$. **5**

Q.4 **a)** State and prove the Cauchy Residue theorem. **10**

b) Attempt **any Two** of the following: **10**

i) Find the image of a straight line $x = 7$ in the complex plane under the transformation $w = e^z$. **5**

ii) Using contour integration, evaluate. $\int_0^{2\pi} \frac{1}{3+\cos\theta} d\theta$. **5**

iii) Use the Argument Principle to evaluate $\int_x \frac{f'(z)}{f(z)} dz$ where $f(z) = \frac{z-2}{z(z-1)(z-2)}$ and x is the circle $|z| = 3$. **5**

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